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On the multiplicative pulsating n-Fibonacci sequence

Alongkot Suvarnamani *

Department of Mathematics, Faculty of Science and Technology, Rajamangala University of Technology Thanyaburi (RMUTT), Pathum Thani, 12110 Thailand

***Corresponding Author:** kotmaster2@rmutt.ac.th

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Abstract

In this paper, we studied the new ideas in generalization of Fibonacci sequences in the case of three sequences. We describe basic concepts that will be used to construct multiplicative pulsating n-Fibonacci sequences of n^{th} order.

Keywords: n-Fibonacci Sequence; Pulsating Fibonacci Sequence; Multiplicative

1. Introduction

The coupled difference equations or recurrence relations are popularized in last decade. They involve two sequences of integers in which the elements of one sequence are part of the generalization of the other, and vice versa. We can say that these are generalization of ordinary recursive sequences and many results can be developed for considering the two sequences are identical.

The concept of coupled Fibonacci sequence was first introduced by Atanassov [1] and also discussed many curious properties and new direction of generalization of Fibonacci sequence in [2 – 5]. They were defined and studied about four different ways to generate coupled sequences and called them 2-Fibonacci sequence (or 2-F sequences). These were new direction of Fibonacci sequence generalizations.

In 2014 Suvarnamani and Jitjang [6] show the result of the multiplicative pulsated 2-Fibonacci sequence. Then Suvarnamani and Koyram [7] proved the new result of the multiplicative pulsated 2-Fibonacci sequence in 2015. After that Suvarnamani and Tatong [8] show the result of the multiplicative pulsated 3-Fibonacci sequence.

In this paper, we studied the new ideas in generalization of Fibonacci sequences in the case of more sequences. We describe basic concepts that will be used to construct multiplicative pulsating 3-Fibonacci sequences of third order. Further, we shall describe fundamental properties.

2. Multiplicative Pulsating n-Fibonacci sequence

Now, a type of Fibonacci-like sequence is introduced. Let $a_1, a_2, a_3, \ldots, a_n$ are fixed real numbers. Let us construct the following n sequences

$$\alpha_{1,0} = a_1, \alpha_{2,0} = a_2, \alpha_{3,0} = a_3, \ldots, \alpha_{n-1,0} = a_{n-1}, \alpha_{n,0} = a_n;$$
$$\alpha_{1,2k+1} = \alpha_{2,2k+1} = \alpha_{3,2k+1} = \ldots = \alpha_{n-1,2k+1} = \alpha_{n,2k+1} = \alpha_{1,2k} \cdot \alpha_{2,2k} \cdot \alpha_{3,2k} \cdot \ldots \cdot \alpha_{n-1,2k} \cdot \alpha_{n,2k};$$

$$\alpha_{1,2k+2} = \alpha_{1,2k+1} \cdot \alpha_{2,2k} \cdot \alpha_{3,2k} \cdot \ldots \cdot \alpha_{n-1,2k} \cdot \alpha_{n,2k};$$
$$\alpha_{2,2k+2} = \alpha_{1,2k} \cdot \alpha_{2,2k+1} \cdot \alpha_{3,2k} \cdot \ldots \cdot \alpha_{n-1,2k} \cdot \alpha_{n,2k};$$
$$\vdots$$
$$\alpha_{n-1,2k+2} = \alpha_{1,2k} \cdot \alpha_{2,2k} \cdot \alpha_{3,2k} \cdot \ldots \cdot \alpha_{n-1,2k+1} \cdot \alpha_{n,2k};$$
$$\alpha_{n,2k+2} = \alpha_{1,2k} \cdot \alpha_{2,2k} \cdot \alpha_{3,2k} \cdot \ldots \cdot \alpha_{n-1,2k} \cdot \alpha_{n,2k+1}.$$

for the non-negative integer k. These n sequences we call a Multiplicative Pulsating n-Fibonacci sequence.

3. Results and Discussion

Now, we shall describe fundamental properties.

Theorem 1. For every non-negative integer k,

$$\alpha_{1,2k} = a_1^{\ x} \cdot a_2^{\ y} \cdot a_3^{\ y} \cdot \ldots \cdot a_n^{\ y};$$
$$\alpha_{2,2k} = a_1^{\ y} \cdot a_2^{\ x} \cdot a_3^{\ y} \cdot \ldots \cdot a_n^{\ y};$$
$$\vdots$$
$$\alpha_{n,2k} = a_1^{\ y} \cdot a_2^{\ y} \cdot a_3^{\ y} \cdot \ldots \cdot a_n^{\ x};$$
$$\alpha_{1,2k+1} = \alpha_{2,2k+1} = \alpha_{3,2k+1} = \ldots = \alpha_{n,2k+1} = a_1^{\ z} \cdot a_2^{\ z} \cdot a_3^{\ z} \cdot \ldots \cdot a_n^{\ z};$$

where $x = \dfrac{(2n-1)^k + (n-1)(-1)^k}{n}$, $y = \dfrac{(2n-1)^k + (-1)^{k+1}}{n}$ and $z = (2n-1)^k$.

Proof. We will prove by mathematical induction.

Let P_k:

$$\alpha_{1,2k} = a_1^{\left(\frac{(2n-1)^k + (n-1)(-1)^k}{n}\right)} \cdot a_2^{\left(\frac{(2n-1)^k + (-1)^{k+1}}{n}\right)} \cdot a_3^{\left(\frac{(2n-1)^k + (-1)^{k+1}}{n}\right)} \cdot \ldots \cdot a_n^{\left(\frac{(2n-1)^k + (-1)^{k+1}}{n}\right)};$$

$$\alpha_{2,2k} = a_1^{\left(\frac{(2n-1)^k + (-1)^{k+1}}{n}\right)} \cdot a_2^{\left(\frac{(2n-1)^k + (n-1)(-1)^k}{n}\right)} \cdot a_3^{\left(\frac{(2n-1)^k + (-1)^{k+1}}{n}\right)} \cdot \ldots \cdot a_n^{\left(\frac{(2n-1)^k + (-1)^{k+1}}{n}\right)};$$

$$\vdots$$

$$\alpha_{n,2k} = a_1^{\left(\frac{(2n-1)^k + (-1)^{k+1}}{n}\right)} \cdot a_2^{\left(\frac{(2n-1)^k + (-1)^{k+1}}{n}\right)} \cdot a_3^{\left(\frac{(2n-1)^k + (-1)^{k+1}}{n}\right)} \cdot \ldots \cdot a_n^{\left(\frac{(2n-1)^k + (n-1)(-1)^k}{n}\right)};$$

$$\alpha_{1,2k+1} = \alpha_{2,2k+1} = \alpha_{3,2k+1} = \ldots = \alpha_{n,2k+1} = a_1^{(2n-1)^k} \cdot a_2^{(2n-1)^k} \cdot a_3^{(2n-1)^k} \cdot \ldots \cdot a_n^{(2n-1)^k}.$$

for every non-negative integer k.

If $k=0$, then

$$\alpha_{1,2(0)} = \alpha_{1,0} = a_1 = a_1^{\left(\frac{(2n-1)^0 + (n-1)(-1)^0}{n}\right)} \cdot a_2^{\left(\frac{(2n-1)^0 + (-1)^{0+1}}{n}\right)} \cdot a_3^{\left(\frac{(2n-1)^0 + (-1)^{0+1}}{n}\right)} \cdot \ldots \cdot a_n^{\left(\frac{(2n-1)^0 + (-1)^{0+1}}{n}\right)};$$

$$\alpha_{2,2(0)} = \alpha_{2,0} = a_2 = a_1^{\left(\frac{(2n-1)^0 + (-1)^{0+1}}{n}\right)} \cdot a_2^{\left(\frac{(2n-1)^0 + (n-1)(-1)^0}{n}\right)} \cdot a_3^{\left(\frac{(2n-1)^0 + (-1)^{0+1}}{n}\right)} \cdot \ldots \cdot a_n^{\left(\frac{(2n-1)^0 + (-1)^{0+1}}{n}\right)};$$

$$\alpha_{3,2(0)} = \alpha_{3,0} = a_3 = a_1^{\left(\frac{(2n-1)^0 + (-1)^{0+1}}{n}\right)} \cdot a_2^{\left(\frac{(2n-1)^0 + (-1)^{0+1}}{n}\right)} \cdot a_3^{\left(\frac{(2n-1)^0 + (n-1)(-1)^0}{n}\right)} \cdot \ldots \cdot a_n^{\left(\frac{(2n-1)^0 + (-1)^{0+1}}{n}\right)};$$

$$\vdots$$

$$\alpha_{n,2(0)} = \alpha_{n,0} = a_n = a_1^{\left(\frac{(2n-1)^0 + (-1)^{0+1}}{n}\right)} \cdot a_2^{\left(\frac{(2n-1)^0 + (-1)^{0+1}}{n}\right)} \cdot a_3^{\left(\frac{(2n-1)^0 + (-1)^{0+1}}{n}\right)} \cdot \ldots \cdot a_n^{\left(\frac{(2n-1)^0 + (n-1)(-1)^0}{n}\right)};$$

$$\alpha_{1,2(0)+1} = \alpha_{2,2(0)+1} = \alpha_{3,2(0)+1} = \ldots = \alpha_{n,2(0)+1} = a_1 \cdot a_2 \cdot a_3 \cdot \ldots \cdot a_n$$
$$= a_1^{(2n-1)^0} \cdot a_2^{(2n-1)^0} \cdot a_3^{(2n-1)^0} \cdot \ldots \cdot a_n^{(2n-1)^0}.$$

Thus P_0 is true.

Next, we assume that P_k is true for some non-negative integer k, i.e.,

$$\alpha_{1,2k} = a_1^{\left(\frac{(2n-1)^k+(n-1)(-1)^k}{n}\right)} \cdot a_2^{\left(\frac{(2n-1)^k+(-1)^{k+1}}{n}\right)} \cdot a_3^{\left(\frac{(2n-1)^k+(-1)^{k+1}}{n}\right)} \cdot \ldots \cdot a_n^{\left(\frac{(2n-1)^k+(-1)^{k+1}}{n}\right)};$$

$$\alpha_{2,2k} = a_1^{\left(\frac{(2n-1)^k+(-1)^{k+1}}{n}\right)} \cdot a_2^{\left(\frac{(2n-1)^k+(n-1)(-1)^k}{n}\right)} \cdot a_3^{\left(\frac{(2n-1)^k+(-1)^{k+1}}{n}\right)} \cdot \ldots \cdot a_n^{\left(\frac{(2n-1)^k+(-1)^{k+1}}{n}\right)};$$

$$\vdots$$

$$\alpha_{n,2k} = a_1^{\left(\frac{(2n-1)^k+(-1)^{k+1}}{n}\right)} \cdot a_2^{\left(\frac{(2n-1)^k+(-1)^{k+1}}{n}\right)} \cdot a_3^{\left(\frac{(2n-1)^k+(-1)^{k+1}}{n}\right)} \cdot \ldots \cdot a_n^{\left(\frac{(2n-1)^k+(n-1)(-1)^k}{n}\right)};$$

$$\alpha_{1,2k+1} = \alpha_{2,2k+1} = \alpha_{3,2k+1} = \ldots = \alpha_{n,2k+1} = a_1^{(2n-1)^k} \cdot a_2^{(2n-1)^k} \cdot a_3^{(2n-1)^k} \cdot \ldots \cdot a_n^{(2n-1)^k}.$$

Then we will show that P_{k+1} is true.

$$\alpha_{1,2(k+1)} = \alpha_{1,2k+2} = \alpha_{1,2k+1} \cdot \alpha_{2,2k} \cdot \alpha_{3,2k} \cdot \ldots \cdot \alpha_{n,2k}$$

$$= \left(\alpha_{1,2k} \cdot \alpha_{2,2k} \cdot \alpha_{3,2k} \cdot \ldots \cdot \alpha_{n,2k}\right) \cdot \alpha_{2,2k} \cdot \alpha_{3,2k} \cdot \ldots \cdot \alpha_{n,2k}$$

$$= a_1^{\left(\frac{(2n-1)^{k+1}+(n-1)(-1)^{k+1}}{n}\right)} \cdot a_2^{\left(\frac{(2n-1)^{k+1}+(-1)^{(k+1)+1}}{n}\right)} \cdot a_3^{\left(\frac{(2n-1)^{k+1}+(-1)^{(k+1)+1}}{n}\right)} \cdot \ldots \cdot a_n^{\left(\frac{(2n-1)^{k+1}+(-1)^{(k+1)+1}}{n}\right)}.$$

$$\alpha_{2,2(k+1)} = \alpha_{2,2k+2} = \alpha_{1,2k} \cdot \alpha_{2,2k+1} \cdot \alpha_{3,2k} \cdot \ldots \cdot \alpha_{n,2k}$$
$$= \alpha_{1,2k} \cdot \left(\alpha_{1,2k} \cdot \alpha_{2,2k} \cdot \alpha_{3,2k} \cdot \ldots \cdot \alpha_{n,2k}\right) \cdot \alpha_{3,2k} \cdot \ldots \cdot \alpha_{n,2k}$$

$$= a_1^{\left(\frac{(2n-1)^{k+1}+(-1)^{(k+1)+1}}{n}\right)} \cdot a_2^{\left(\frac{(2n-1)^{k+1}+(n-1)(-1)^{k+1}}{n}\right)} \cdot a_3^{\left(\frac{(2n-1)^{k+1}+(-1)^{(k+1)+1}}{n}\right)} \cdot \ldots \cdot a_n^{\left(\frac{(2n-1)^{k+1}+(-1)^{(k+1)+1}}{n}\right)}.$$

$$\alpha_{3,2(k+1)} = \alpha_{3,2k+2} = \alpha_{1,2k} \cdot \alpha_{2,2k} \cdot \alpha_{3,2k+1} \cdot \ldots \cdot \alpha_{n,2k}$$
$$= \alpha_{1,2k} \cdot \alpha_{2,2k} \cdot \left(\alpha_{1,2k} \cdot \alpha_{2,2k} \cdot \alpha_{3,2k} \cdot \ldots \cdot \alpha_{n,2k}\right) \cdot \ldots \cdot \alpha_{n,2k}$$
$$= a_1^{\left(\frac{(2n-1)^{k+1} + (-1)^{(k+1)+1}}{n}\right)} \cdot a_2^{\left(\frac{(2n-1)^{k+1} + (-1)^{(k+1)+1}}{n}\right)} \cdot a_3^{\left(\frac{(2n-1)^{k+1} + (n-1)(-1)^{k+1}}{n}\right)} \cdot \ldots \cdot a_n^{\left(\frac{(2n-1)^{k+1} + (-1)^{(k+1)+1}}{n}\right)}.$$

$$\vdots$$

$$\alpha_{n,2(k+1)} = \alpha_{n,2k+2} = \alpha_{1,2k} \cdot \alpha_{2,2k} \cdot \alpha_{3,2k} \cdot \ldots \cdot \alpha_{n,2k+1}$$
$$= \alpha_{1,2k} \cdot \alpha_{2,2k} \cdot \alpha_{3,2k} \cdot \ldots \cdot \left(\alpha_{1,2k} \cdot \alpha_{2,2k} \cdot \alpha_{3,2k} \cdot \ldots \cdot \alpha_{n,2k}\right)$$
$$= a_1^{\left(\frac{(2n-1)^{k+1} + (-1)^{(k+1)+1}}{n}\right)} \cdot a_2^{\left(\frac{(2n-1)^{k+1} + (-1)^{(k+1)+1}}{n}\right)} \cdot a_3^{\left(\frac{(2n-1)^{k+1} + (-1)^{(k+1)+1}}{n}\right)} \cdot \ldots \cdot a_n^{\left(\frac{(2n-1)^{k+1} + (n-1)(-1)^{k+1}}{n}\right)}.$$

$$\alpha_{1,2(k+1)+1} = \alpha_{2,2(k+1)+1} = \alpha_{3,2(k+1)+1} = \ldots = \alpha_{n,2(k+1)+1} = \alpha_{1,2k+3}$$
$$= \alpha_{1,2k+2} \cdot \alpha_{2,2k+2} \cdot \alpha_{3,2k+2} \cdot \ldots \cdot \alpha_{n,2k+2}$$
$$= a_1^{(2n-1)^{k+1}} \cdot a_2^{(2n-1)^{k+1}} \cdot a_3^{(2n-1)^{k+1}} \cdot \ldots \cdot a_n^{(2n-1)^{k+1}}.$$

So, P_{k+1} is true.

By mathematical induction, P_k is true for all non-negative integer k.

Colloraly 2. For every non-negative integer k,

$$\alpha_{1,2k} = a_1^{\left(\frac{3^k + 3(-1)^k}{2}\right)} \cdot a_2^{\left(\frac{3^k + (-1)^{k+1}}{2}\right)};$$

$$\alpha_{2,2k} = a_1^{\left(\frac{3^k + (-1)^{k+1}}{2}\right)} \cdot a_2^{\left(\frac{3^k + (-1)^k}{2}\right)};$$

$$\alpha_{1,2k+1} = \alpha_{2,2k+1} = a_1^{3^k} \cdot a_2^{3^k}.$$

Proof. From Theorem 1, if $n=2$ then we get

$$\alpha_{1,2k}=a_1^{\left(\frac{3^k+(-1)^k}{2}\right)}\cdot a_2^{\left(\frac{3^k+(-1)^{k+1}}{2}\right)};$$

$$\alpha_{2,2k}=a_1^{\left(\frac{3^k+(-1)^{k+1}}{2}\right)}\cdot a_2^{\left(\frac{3^k+(-1)^k}{2}\right)};$$

$$\alpha_{1,2k+1}=\alpha_{2,2k+1}=a_1^{3^k}\cdot a_2^{3^k}.$$

So, we get the result which showed in [7].

Colloraly 3. For every non-negative integer k,

$$\alpha_{1,2k}=a_1^{\left(\frac{5^k+2(-1)^k}{3}\right)}\cdot a_2^{\left(\frac{5^k+(-1)^{k+1}}{3}\right)}\cdot a_3^{\left(\frac{5^k+(-1)^{k+1}}{3}\right)};$$

$$\alpha_{2,2k}=a_1^{\left(\frac{5^k+(-1)^{k+1}}{3}\right)}\cdot a_2^{\left(\frac{5^k+2(-1)^k}{3}\right)}\cdot a_3^{\left(\frac{5^k+(-1)^{k+1}}{3}\right)};$$

$$\alpha_{3,2k}=a_1^{\left(\frac{5^k+(-1)^{k+1}}{3}\right)}\cdot a_2^{\left(\frac{5^k+(-1)^{k+1}}{3}\right)}\cdot a_3^{\left(\frac{5^k+2(-1)^k}{3}\right)};$$

$$\alpha_{1,2k+1}=\alpha_{2,2k+1}=\alpha_{3,2k+1}=a_1^{5^k}\cdot a_2^{5^k}\cdot a_3^{5^k}.$$

Proof. From Theorem 3.1, if $n=3$ then we get

$$\alpha_{1,2k}=a_1^{\left(\frac{5^k+2(-1)^k}{3}\right)}\cdot a_2^{\left(\frac{5^k+(-1)^{k+1}}{3}\right)}\cdot a_3^{\left(\frac{5^k+(-1)^{k+1}}{3}\right)};$$

$$\alpha_{2,2k}=a_1^{\left(\frac{5^k+(-1)^{k+1}}{3}\right)}\cdot a_2^{\left(\frac{5^k+2(-1)^k}{3}\right)}\cdot a_3^{\left(\frac{5^k+(-1)^{k+1}}{3}\right)};$$

$$\alpha_{3,2k}=a_1^{\left(\frac{5^k+(-1)^{k+1}}{3}\right)}\cdot a_2^{\left(\frac{5^k+(-1)^{k+1}}{3}\right)}\cdot a_3^{\left(\frac{5^k+2(-1)^k}{3}\right)};$$

$$\alpha_{1,2k+1}=\alpha_{2,2k+1}=\alpha_{3,2k+1}=a_1^{5^k}\cdot a_2^{5^k}\cdot a_3^{5^k}.$$

So, we get the result which showed in [8].

4. Conclusion

In this study, we show the proof of the explicit formulas of multiplicative pulsating n-Fibonacci sequences of n^{th} order. That is

$$\alpha_{1,2k} = a_1^{\left(\frac{(2n-1)^k+(n-1)(-1)^k}{n}\right)} \cdot a_2^{\left(\frac{(2n-1)^k+(-1)^{k+1}}{n}\right)} \cdot a_3^{\left(\frac{(2n-1)^k+(-1)^{k+1}}{n}\right)} \cdot \ldots \cdot a_n^{\left(\frac{(2n-1)^k+(-1)^{k+1}}{n}\right)};$$

$$\alpha_{2,2k} = a_1^{\left(\frac{(2n-1)^k+(-1)^{k+1}}{n}\right)} \cdot a_2^{\left(\frac{(2n-1)^k+(n-1)(-1)^k}{n}\right)} \cdot a_3^{\left(\frac{(2n-1)^k+(-1)^{k+1}}{n}\right)} \cdot \ldots \cdot a_n^{\left(\frac{(2n-1)^k+(-1)^{k+1}}{n}\right)};$$

$$\vdots$$

$$\alpha_{n,2k} = a_1^{\left(\frac{(2n-1)^k+(-1)^{k+1}}{n}\right)} \cdot a_2^{\left(\frac{(2n-1)^k+(-1)^{k+1}}{n}\right)} \cdot a_3^{\left(\frac{(2n-1)^k+(-1)^{k+1}}{n}\right)} \cdot \ldots \cdot a_n^{\left(\frac{(2n-1)^k+(n-1)(-1)^k}{n}\right)};$$

$$\alpha_{1,2k+1} = \alpha_{2,2k+1} = \alpha_{3,2k+1} = \ldots = \alpha_{n,2k+1} = a_1^{(2n-1)^k} \cdot a_2^{(2n-1)^k} \cdot a_3^{(2n-1)^k} \cdot \ldots \cdot a_n^{(2n-1)^k}$$

for every non-negative integer k.

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